The Astrophysical Journal, 885:148 (13pp), 2019 November 10 https://doi.org/10.3847/1538-4357/ab48e0

The Long-standing Closure Crisis in Coronal Plasmas

J. D. Scudder
Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242, USA
Received 2019 July 4; revised 2019 September 25; accepted 2019 September 26; published 2019 November 11

Abstract

Coronal and solar wind physics have long used plasma fluid models to motivate physical explanations of observations; the hypothesized model is introduced into a fluid simulation to see if observations are reproduced. This procedure is called Verification of Mechanism (VoM) modeling; it is contingent on the self consistency of the closure that made the simulation possible. Inner corona VoMs typically assume weak gradient Spitzer–Braginskii closures. Four prominent coronal VoMs in place for decades are shown to contradict their closure hypotheses, demonstrably shaping coronal and solar wind research. These findings have been possible since 1953. This unchallenged evolution is worth understanding, so that similarly flawed VoMs do not continue to mislead new research. As a first step in this direction, this paper organizes four a posteriori quantitative tests for the purpose of easily screening the physical integrity of a proposed VoM. A fifth screen involving the thermal force, the tandem of the heat flux, has been shown to be mandatory when VoMs involve species-specific energy equations. VoM modeling will soon be required to advance *Parker Solar Probe* and *Solar Orbiter* science. Such modeling cannot advance the physical understanding sought by these missions unless the closures adopted (i) are demonstrated to be self consistent for the VoM plasma Knudsen numbers, (ii) are verified a posteriori as possessing nonnegative VDFs throughout the simulated volume, and (iii) include the physical completeness of thermal force physics when the VoM requires species-specific energy equations.

Unified Astronomy Thesaurus concepts: Solar wind (1534); Solar corona (1483); Astrophysical fluid dynamics (101); Solar coronal heating (1989)

1. The Problem

Identifying the precise mechanism essential for the formation of the solar corona and the extreme states of the solar wind expansion are the central objectives of NASA's recently launched *Parker Solar Probe* and ESA's soon to be launched *Solar Orbiter* spacecraft. These bold scientific missions seek to screen for *the physical* mechanism (s) necessary to self-consistently explain the apparently common circumstances for the existence of these key regions in our star (and others). The first rung of such explanations would be a physically defensible VoM model that demonstrates cause and effect relationships of the physical mechanism and the observed morphology being explained. The closed moment plasma fluid equations are invariably used to demonstrate the behavior of the causal mechanism and its observational consequences.

For the VoM demonstration to be useful, such a VoM must be free from concerns that the fluid equations are closed improperly when being used to demonstrate the mechanism's role in causing the observed signatures. As an example, if the VoM involved the effects of a steepening wave train, a possible concern might be violations of the weak gradient premises of the closure in the steeper parts of the train in the fluid solution offered. In this way it is possible that closures may be self consistent for the spatial locale without the suggested mechanism, but no longer adequate for the purposes of evaluating the mechanism's effects in the same region.

Central to realizing this objective is producing VoM models that are based on a consistent set of physical first principles with negligible reliance on ad hoc or adjustable *free parameters, or bridging functions* to enhance model data agreement. Because of the wide range of macroscopic conditions involved in the low corona and wind acceleration region, these VoMs invariably use quasi-neutral fluid scale models for the plasma.

Viable fluid models for a plasma rely on the validity of delicate averaging and other approximations called *closure approaches*; they attempt to approximate the full physical description of the plasma, hoping to avoid the technical difficulties of attempting a full kinetic model following the evolution and interactions of electrons, ions and the underlying electro-magnetic field that may not even be computationally tractable. It is *generally* unknown whether consistent closures can be found for astrophysical systems with large variations of their macroscopic parameters; consistency can only be demonstrated by a posterori validation of the solutions produced by the closure of the type discussed below (CJC).

Large scale plasma fluid modeling presumes the maintenance of quasi-neutrality; the intrinsic inhomogeneities of astrophysics induced by gravity, rotation and radiation require substantial parallel electric fields $E_{\|}$ as part of any steady spatial equilibria. These plasmas are different from those in Local Thermodynamic Equilibrium (LTE) which have no steady-state $E_{\|}$. Accordingly, the closure description of these fluid plasmas may well be different from those described as being only slightly removed from LTE. Nonetheless, at present *most* astrophysical fluid plasma closure recipes used for VoMs are borrowed from derivations made for other plasma systems infinitesimally displaced from LTE; closure recipes associated with the names of Spitzer & Härm (1953) and Braginskii (1965) are of this near LTE type.

Fluid models can be very informative and attractive since they are computationally less demanding than kinetic descriptions. A *relevant, validated* closure approach enables this efficiency. A *relevant* closure approach is usually framed for the system at hand and is likely not *relevant* when borrowed from another system. While it is common to use the same fluid closure for the entire problem at hand, it is not known ab initio if this approach will be consistent across the entire modeled problem; approximations valid in one regime may collapse before the spatial volume being modeled is traversed. This consideration is an ever present worry in plasma fluid scale plasma modeling and thus, VoMs. The only known antidote for this problem is a posteriori validation, a process discussed below and labeled Completely Justified Closure (CJC).

The closure approach for a system is a process, not a formulaic panacea for the kinetic complications being avoided. One *tentative* outcome of the closure process is a mathematically rigorous derivation of the *closure rule* (CR); this rule (if derived for the system at hand) assumes enough things are true about the behavior of the "yet to be found fluid solutions" to allow the *conditional* replacement of the kinetic descriptions with a finite number of fluid equations for the plasma fluid moments. While a BCR or CR does have a formulaic character, the closure approach does not terminate here.

The CR is tentative, derived by leveraging one or more assumptions about the properties of the *still unknown fluid solutions of the system under study*. The fluid solutions found with the tentative CR have no value for the VoM at hand unless they are "validated" a posteriori to be consistent with the assumptions made to derive the CR. Addressing all parts of the last sentence is the definition of the CJC approach for the *specific* problem at hand.

Below we use the concept of CJC as different from the CR; CJC implies that the "hypotheses-closure-rule-retro-validation" chain has been completed for the specific problem at hand. By contrast, the typical VoM in the present literature uses the adopted or borrowed CR as the only recipe for avoiding solutions of the kinetic equations. As the examples below show, the CJC retro-validation of the CR have never been considered; the hypothesized properties of the solution enabled by the closure are never verified as being consistent with the suppositions that enabled the "rigor" of the CR's derivation.

Nonetheless, using the CR alone the fluid solutions can be found and are usually internally inconsistent. Surprising, but true, these flawed solutions possess no visibly untoward properties; nonetheless, upon asking the right questions of their smooth profiles they usually contradict the presumptions that allowed the CR to be advanced in the first place.

The minimal set of CJC physical hurdles to be surmounted by any VoM model involve demonstrating that the CR has not allowed a variation in the fluid solutions that was precluded by the reasoning assumed when advancing the CR. Being a minimal set, passing the CJC hurdles does not guarantee that the model problem's fluid solution for the VoM is totally consistent with what the *unknown* kinetic solution would have produced.

The CJC hurdles serve to *screen* against relying on solutions with rudimentary violations of mathematical and physical consistency. The first two of the set below involve *mathematical consistency* of the typically *perturbative* CR formulations: (i) are its results consistently perturbative as assumed by the CR? (ii) is the approach convergent? The second two hurdles involve *physical consistency* at a very rudimentary level: (iii) does the underlying VDF premise of the closure remain *essentially nonnegative definite* across the fluid solution, and if accessible (iv) do the observed VDF emulate the VDF that underlie the closure? The mathematical tests speak to perturbation rigor expected by the CR; the physical tests about nonnegativity of the VDF speak to the viability of the VDF function space for the kinetic description as approximated. These conditions are deeply related; demonstration that *any* of the four hurdles are violated implies the CR is vacated for the system inventoried. In this sense these hurdles serve as *vetoes* of CR proposals and represent a minimalist check on the integrity of the VoM.

The absence of the thermal force in the fluid equations when different energy equations are required in the VoM serves as another veto: essential incompleteness of the plasma description. When a plasma supports heat conduction it also supports a tandem thermal force that together allows steady heat to flow without electrical current. In this circumstance the thermal force is the enforcer of charge neutrality as part of the Generalized Ohm's Law (Spitzer & Härm 1953; Braginskii 1965; Rossi & Olbert 1970; Fitzpatrick 2014; Scudder 2019a).

This paper looks carefully at 4 prominent VoMs in the solar corona-wind literature; all are vacated by being internally contradicted when attempting CJC. Distributed over nearly 80 yr these studies suggest fluid VoMs need to proceed with CJC closures, before their solutions become candidates for the physical assay of the system under study. While clearing CJC hurdles are necessary for establishing candidate physical explanations, they should be viewed as plausible, rather than established candidates for VoM, pending further detailed kinetic modeling that might be performed that is independent of the need of a CR.

1.1. Paper Plan

Section 2 clarifies the two parts of the "closure approach", and gives brief introductions to the two main ways that fluid, CRs, have been introduced to plasma problems. Section 3 discusses four approaches to completely justify the closure approach CJC by "validating" the hypotheses that allowed the CR to be suggested. Section 4 retrospectively re-examines four well established coronal wind case studies that span 80 yr in the literature, using the tests of Section 3. Section 5 inventories the impact of finding that these well established results are vacated by their failure of CJC validation. The section is closed by a brief discussion of the challenges these failures represent for fulfilling the goals of the *Parker Solar Probe* and *Solar Orbiter missions*. Appendix A lists acronyms used and where they are first defined. Appendix B describes the construction of Figure of Merit (FoM) values that screen for unphysically negative VDFs that can occur at finite Knudsen numbers under all Closure Rules with a VDF predicate.

2. Closure: Part(I) CR

Plasma closure developments have been strongly shaped by those first developed for neutral gases. The standard approach for deriving such closures is to presume that randomization by collisions occurs over scales $\lambda_{\rm mfp}$ much shorter than any background gradient scales, L. A common Chapman–Enskog (CE; Chapman 1916, Enskog 1917) closure calculation is an expansion about LTE; it assumes a perturbatively small *Knudsen number*, $\mathbb{K} \equiv \lambda(w*)/ L < 1$, where the mean free path $\lambda(w*)$ used is that of the particle with the thermal speed w^*. A less extensively used Grad N-moment method

(Grad 1949) uses trial VDF functions dictated by the order N of the number of moments treated equally in their evolution, attempting to describe systems with non-perturbatively finite $\mathbb{K}$.

Both CE and Grad CRs involve velocity space functional expansions for the VDFs that are the kinetic basis of the CR. The physical VDF is always nonnegative; however, these functional expansions for the VDF are *not* guaranteed to remain sufficiently nonnegative as the $\mathbb{K}$ increases. For very weak $\mathbb{K}$, these negative values occur at speeds that are large compared to the thermal speed; as the $\mathbb{K}$ grows, this problem moves to lower speeds, making the approximation to the VDF negative in regimes that unphysically impact moments retained in the subsequent fluid equations.

Typically, astrophysical plasmas are characterized by strong $\mathbb{K}$ variations, starting very small near, but below, the stellar surface and rapidly becoming non-perturbative by 1.05 stellar radii from the center of the star (Scudder & Karimabadi 2013). When starting with a LTE CR, the plasma extensions with altitude rapidly have finite $\mathbb{K}$ with ever encroaching velocity space domains that are negative when using the LTE basis functions. This type of failure and searching for its occurrence is part of the vigilance required when using CRs in fluid-based VoMs in the inner corona region.

Using these CRs with prominently negative VDFs is one type of hidden problem that *can* remain undetected if CJC checking is not performed. Since the physical VDF is nonnegative definite, the speed regime where this negativity occurs must be monitored enroute to judging the spatial domain of success of the closure for a given problem; an approach to such documentations using FoMs is discussed in Section 3 and in Appendix B.

2.1. Think Perturbative: Chapman–Enskog (CE)

Considering the small Knudsen ($\mathbb{K} < 1$) regime leads to thinking of the transport effects as being described as a perturbative velocity space correction to a lowest order Maxwellian VDF with uniform temperature and density that would typify LTE with $\mathbb{K}(w) = 0$, at finite density. This would be a spatially uniform state, requiring no $E_\|$ to enforce quasi-neutrality. In this regime, the lowest rung of the perturbation starts with an initial Maxwellian that zeros the collision operator and is consistent with no gradients driving the kinetic equation. The gradients allowed in macroscopic variables are *all* assumed perturbative relative to the presumed uniform underlying conditions for the LTE plasma. Closures associated with the names (Chapman 1916) and (Enskog (1917)) have led to the frequently used Fourier law form for the isobaric *plasma* CR

$$q_\| = -\hat{\boldsymbol{b}} \cdot \mathcal{K}(T) \nabla \cdot \boldsymbol{T}, \tag{1}$$

associated with the names Spitzer and Braginskii (Spitzer 1962; Braginskii 1965); this formula has been generalized for *perturbative* pressure gradients (Ferziger & Kaper 1972; Balescu 1988). By the assumptions of their derivations, this and similar approaches yield *perturbative, weak gradient and, thus, weak* $\mathbb{K}$ CR. The reference scale L used for the Knudsen perturbation ordering parameter $\mathbb{K}$ in the CE formulation is the *shortest* macroscopic scale along the magnetic field of the solution's n, P, U profiles (see paragraph preceding Equation (1.25) Braginskii 1965); in astrophysical plasmas, this will usually not be the scale of the temperature.

Brief mention should be made of the spillover of transport arguments from collisional gas dynamics into those for a plasma. The use above of one number, $\mathbb{K}$, to gauge the allowable size of the velocity space corrections for the *entire random spread of w's in the VDF* incorrectly assumes that the plasma scattering physics emulates that of the neutral gas. For neutral gases, $\lambda(w)$ is virtually independent of the relative speed of the participants so that the Knudsen number at any speed in the neutral gas and the velocity space average of it, $\langle\mathbb{K}\rangle$, are virtually interchangeable:

$$\langle \mathbb{K}^g \rangle \simeq \mathbb{K}^g(w). \tag{2}$$

For such gases, small $\mathbb{K}^g(w*)$ implies small $\mathbb{K}^g(w)$ for essentially all speeds w different from $w*$ in the velocity space. Thus, one number can become the small parameter for the corrections to the VDF at all speeds in the neutral gas. For convenience, it is taken to be evaluated at the thermal speed, $w*$.

By contrast, small $\mathbb{K}^p(w*)$ in a plasma does not imply $\mathbb{K}^p(w)$ remains sufficiently small for all random speeds, w, unless $\mathbb{K}^p(w*)$ is *very* small. The issue is that for a plasma

$$\langle \mathbb{K}^p \rangle \neq \mathbb{K}^p(w);$$
$$\mathbb{K}^p(w) \simeq \mathbb{K}^p(w*)\left(\frac{w}{w*}\right)^4, \tag{3}$$

where $w*$ is a reference speed in the plasma, usually the thermal speed or rms thermal speed, depending on the application. This has the attention grabbing implication that a perturbative Spitzer–Braginskii calculation for a plasma requires a much smaller $\mathbb{K}^p(w*) < 0.01$ (Gurevitch & Isotomin 1979; Gray & Kilkenny 1980; Schoub 1983; Scudder & Olbert 1983) than for a neutral gas, where $\mathbb{K}^g(w*) < 1$ usually suffices. Many who have integrated the plasma fluid description with Spitzer CR have erred in this gas dynamical way, trusting Spitzer's formulation out to regimes where $<\mathbb{K}^p(w*)> \simeq 1$ (Hartle & Sturrock 1968; Mikic et al. 1999; Cranmer et al. 2007; Chandran et al. 2011; Bale et al. 2013; van der Holst et al. 2014; Gombosi et al. 2018).

While Spitzer's Fourier heat law appears to only depend on the temperature gradient, this does not mean it would still have that form if other macroscopic gradients of the solution had shorter scales than that for T_e. The essential reason for this is that the CE perturbation's ordering assumes that all important contributors to intermediate results (like zero current) are properly reconciled at that order when the final result is stated. It is the accident of expanding about a Maxwellian in otherwise homogeneous system with a weak temperature gradient that the heat flux only depends on L_{T_e} (e.g., Equation (35) Scudder & Olbert 1983).

The scale L in the relevant Knudsen number, $\mathbb{K}_\chi$, is determined by the field aligned logarithmic gradient, or largest inverse scale (for $\chi = \{n, u, P\}$) determined by

$$L_\chi^{-1} = |\hat{\boldsymbol{b}} \cdot \nabla \ln \chi|. \tag{4}$$

Since L_χ can only be determined a posteriori after supposing a CR and generating moment fluid profiles of the solution, the validity of the closure approach and its enabled solution are unknown until a retrospective CJC survey of the solution is conducted; if successful, this survey should show that the

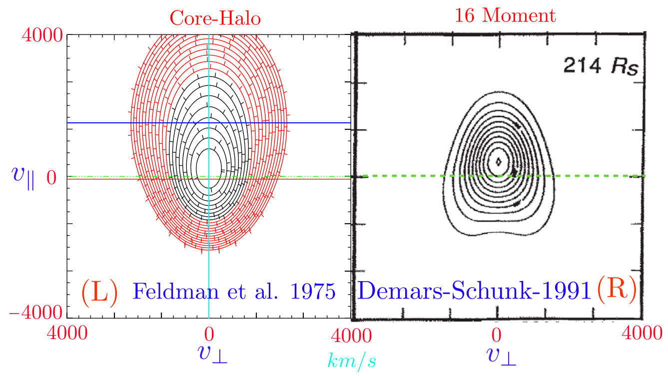

Figure 1. (L) Core-Halo eVDF with *same fixed moments* $[n, T_{\|,e}, T_{\perp,e}, q_{\|}]$ as in (R) set for a 16-moment model of the solar wind. (L) two displaced nearly elliptic zones of the VDF with different slopes of core-halo distribution, while (R) is concentric ellipses and gradually skewed.

shortest local L_χ and the variation of $\mathbb{K}_\chi$ throughout the solution remain sufficiently small for the CR's transport perturbation theory to be convergent.

A source of considerable confusion arises if one attempts to export CRs across regimes that differ from those where the CR is CJC verified. As an example, Spitzer and Braginskii results were derived assuming no background pressure gradient or external forces were factors in the equilibrium (Spitzer & Härm 1953). This example demonstrates the context dependence of CJC: when the scales of electron pressure were infinite and $\mathbb{K}_T < .01$ Spitzer–Härm enjoys CJC; for the same $\mathbb{K}_T$ in the presence of externally supported astrophysical pressure gradients Spitzer's formulation is *no longer CJC* consistent. In astrophysical plasmas, like the corona, the density gradient and thus that of the pressure have scales considerably shorter than the temperature gradients, making this second circumstance particularly common.

In the case of Spitzer's formulation, since 1983, this means its CJC defense requires that the VoM profiles satisfy $\mathbb{K}_{P_e} < .01$ (Gurevitch & Isotomin 1979; Gray & Kilkenny 1980; Schoub 1983; Scudder & Olbert 1983), where

$$\mathbb{K}_{P_e} \equiv |\hat{\boldsymbol{b}} \cdot \nabla \ln P_e| \lambda_{\rm mfp}(w*), \quad (5)$$

where the usual precedence in astrophysics of pressure gradients than temperature gradients has been assumed. In Spitzer–Härm where there was no density gradient, the condition above reduces to an electron temperature Knudsen number condition.

Such a posterori inventories are the only real measure of the CE hypotheses, or the physical worthiness of the VoM solutions attempted. Unfortunately, such analysis is rarely discussed, and, if done, virtually never published.

Quite commonly, coronal and wind modelers will estimate the size of $\mathbb{K}_{T_e}$ using the medium's longer temperature scale L_{T_e} when assessing their "safety" when using Spitzer's Fourier heat law for closure (e.g., Bale et al. 2013), ignoring the presence of the much shorter field aligned L_{P_e}. If this error were compounded by enforcing a condition like $\mathbb{K}_{T_e} < 1$ rather than $\mathbb{K}_{T_e} < 0.01$ Spitzer's heat law is invalidated much earlier than erroneously inferred, contracting the domain of validity of the VoM modeling, potentially precluding the desired spatial domain. In the solar wind $L_{T_e}/L_{P_e} \simeq 5$–20, (where the smaller number is near 1 au, and the larger one near the base of the corona), so that making both errors simultaneously would misjudge the system's limiting K_{T_e} to be 500–2000 times larger than it should be based on the mathematical consistency required by these perturbative transport expansions.

2.2. Think Broadly: Grad's N-moment Method

The *N*-moment method is a different transport closure approach initiated by Grad (1949) and developed for planetary aeronomy and space applications (Demars & Schunk 1991; Li 1999; Lie-Svendsen et al. 2001). It attempts to alleviate the perturbative limitation of CE closure by treating *N*-moments more equally than just n, U, T that occur in the LTE Maxwellian distribution. The size of N indexes the depth in the moment hierarchy included. For solar wind modeling, $N = 13$ and 16 versions of this approach have been published that elevate the heat flux tensor elements to one of these "more equal" moments. In principle, the moments above the temperature but below N are no longer required to be perturbative corrections as is presumed in CE.

Achieving closure at level N, given an assumed lowest order distribution (weighting function) f^0, is accomplished by a rigid recipe for the underlying form for the VDF that supports the transport and automatically gives a mathematical closure. These functions have the form

$$f_0(n, T, U)(1 + \Phi(D + v_i g_i + v_i v_j K_{ij} + v_i v_j v_k Q_{ijk} + ...)), \quad (6)$$

where the Einstein summation convention for repeated indices is implied. The correction terms in Equation (6) are a series of completely contracted tensorial terms that make Φ a scalar, but where the values of the actual tensor elements are combinations of the moments of the unknown distribution function that are below the highest moment level retained, N. As N increases, more multinomial tensor contractions are involved in Φ. The presence of these high "powers" of v in Φ can ultimately have consequences that the corrected VDF can become negative if the associated, spatially varying moments become too large (see below and Appendix B).

The physical meaning of this approach to mathematical truncation is not clear, but always leads to a CR of some complexity and generally stiff differential equations. The hope is this approach can permit solutions to the transport problem to be convergent with not such strict requirements on $\mathbb{K}$. Intuition is sparse showing how to pick appropriate N values to afford suitable approximations for given problems, as is the connection between N and the magnitude of relief on the bounds for $\mathbb{K}$ found with CE. In principle, f_0 is free to be chosen but has been kept Maxwellian or Bi-Maxwellian in the solar wind modeling.

The size of N is a *choice* that must be high enough to include the moments thought to be important; from this choice, the underlying VDF function space implied for the CR is prescribed given f^0. This rigidity may make any given truncation still unsuitable if the microphysics to be described cannot be supported in the space of functions allowed by N and f^0; this mismatch is also in tension with the physical requirement that the VDF remain nonnegative. This is a troublesome aspect of the *N*-moment approach, since it is well known that a given finite set of moments can be replicated by many different distribution functions. Including more moments for the same f^0 still may not address the underlying needed behavior required by the Boltzmann equation.

The N-moment method is thought to be more flexible than CE, at the penalty of requiring more moment boundary conditions than may be experimentally constrained. It is unknown whether raising N is a more expeditious route to better solutions for finite $\mathbb{K}$ than changing the assumed form for f^0.

The $N = 16$ expansion about the Maxwellian f^0 shown in Equation (6) is given explicitly by Equations (A1)–(A3) in (Demars & Schunk 1991); electron eVDFs between 1 au to 28 $R_\odot$, were summarized (see Figure 1(k)). For a 1 au observer, Figure 1 contrasts the published 16-moment electron phase space portraits (R) with those routinely seen in the solar wind (L), e.g., Feldman et al. (1975). Both insets have the same density, heat flux, pressure anisotropy, and temperatures. The black contours in (L) are those that subtend the extremes of contours published for the 16-moment eVDF (R). The red contours (L) highlight the routinely larger velocity space spread of the phase space in nature versus that allowed by this particular 16-moment method (R).

The core-halo partial pressure and density partition were not prescribed by the 16-moment method; thus, there is considerable freedom in the core-halo phase space density, even while matching the heat flux and overall pressure anisotropy. For this illustration, the core-halo temperature fraction was chosen as the typical 1 au values for these conditions. However, by arbitrarily adjusting this ratio, the VDF (L) can be made to look like the one on the (R). This flexibility demonstrates that the 16-moment method can miss the microstate while connecting the Knudsen numbers of the flow. In this sense, it is not fully predictive of underlying effects not far removed from the heat flux as the last moment retained. Similarly, the pressure anisotropy in the core-halo phase space *could* be re-partitioned differently between core and halo contributions to give further mismatches between a realizable core-halo VDF and the only VDF available by this 16-moment method at this location for the stated f^0.

These juxtaposed eVDF panels highlight the very real possibility that N-moment closure attempts to reach to higher Knudsen numbers are not guaranteed to achieve the behavior that the underlying kinetic equation would have achieved. Matching theoretical and empirical moments *need not* imply the underlying physics of the VDFs and collisions are treated as the kinetic equation would have shown. Conversely, *it is hard* a priori *to know what value for N might* contain *all* of the essential properties of the astrophysical problem.

3. Closure Part (II): CJC

A fluid CR represents a hypothesis. This hypothesis is only corroborated after validation, CJC, shows that the enabled fluid solution for the physical system under study meets the presumed properties that allowed the CR to be derived.

Established fluid paradigms in coronal physics have usually been argued in the framework of the CR formulated by Spitzer–Braginskii. Even very recent constructions of VoMs use this pedigreed approach in the inner heliosphere below 5 $R_\odot$. Multiple practitioners currently use Spitzer's heat conduction CR to model the formation of the corona past the temperature maximum and across the vast majority of the wind's acceleration out to the sonic point ($\simeq 5\,R_\odot$), switching to another heat flux formulation for larger radial distances. Accordingly, there are numerous results about the plasmas below the sonic point that are at risk if Spitzer's CR fails CJC.

There are four, related, part (II), closure validation tests as part of CJC, that could have been, but were not, made by those who have used Spitzer's formulation in coronal and solar wind VoMs.

(i) *The CE framework involves a consistent use of perturbation theory.* Since 1953, it has been known that Spitzer's formulation (Spitzer & Härm 1953; Spitzer 1962), as a representative of the CE type, requires a specific *perturbative* $E_\parallel$ that is already built into their proposed heat law CR; it has the form

$$eE_\parallel = -0.71\hat{\boldsymbol{b}} \cdot \nabla(kT_e)$$
$$\mathbb{E}_\parallel = 0.355\mathbb{K}_{T_e}$$
$$\mathbb{E}_\parallel \equiv \frac{E_\parallel}{E_D}, \tag{7}$$

where Dreicer's electric field, E_D, defined by $eE_D\lambda_{\rm mfp} \equiv 2kT_e$ and $\lambda_{\rm mfp}$ is the coulomb mean free path for the thermal speed electron. This electric field was required to preempt parallel thermo-electrical currents that otherwise flow in the presence of the proposed heat flow SH (Spitzer & Härm 1953; Spitzer 1962). The second form in Equation (7) is the original SH relationship scaled by Dreicer's electric field followed by using the definition of the electron temperature's Knudsen number.

The plasma pressure profile of the VoM *solution* implies the presence of an $E_\parallel$ in the modeled fluid solution for the plasma; *if* the profile were isothermal, such pressure gradients support the Pannekoek–Rosseland parallel electric field required for quasi-neutrality and equal scale heights for electrons and protons in stellar atmospheres (see Pannekoek 1922; Rosseland 1924). From conservation laws, $E_\parallel$ more generally has a lower limit (Scudder 2019a) based on the Generalized Ohm's Law (Rossi & Olbert 1970) of

$$\mathbb{E}_\parallel \geqslant \mathbb{K}_{P_e,\parallel}/2, \tag{8}$$

where, by its definition

$$\mathbb{K}_{P_e,\parallel} \equiv \mathbb{K}_{n,\parallel} + \mathbb{K}_{T_e,\parallel} \tag{9}$$

and

$$\mathbb{K}_{P_e,\parallel} = -\lambda_{\rm mfp}\hat{\boldsymbol{b}} \cdot \nabla lnP_e. \tag{10}$$

If, as is common, the density and temperature gradients are aligned, the magnitude of $\mathbb{K}_{P_e}$ always exceeds that of $\mathbb{K}_{T_e}$. Since the density gradients are usually steeper than those of T_e even when their senses are opposed, as occurs just above the photosphere, one generally finds for astrophysical profiles that

$$|\mathbb{K}_{P_e,\parallel}| > 0.71|\mathbb{K}_{T_e,\parallel}|. \tag{11}$$

As shown below with the relatively recent solutions (Cranmer et al. 2007; Chandran et al. 2011) describing innermost regions with T and *n anticorrelated*, Equation (11) is nonetheless satisfied (see Figure 4).

Any VoM profile obtained with Spitzer closure that satisfies Equation (11) has used Spitzer's closure in an inappropriately inhomogeneous system. Any time after 1953, the use of Spitzer's closure for a plasma could have been screened a posteriori for inappropriate use by contrasting the two profiles in Equation (11). We show below that the conclusions of each of our four studies are invalidated by this simple test.

(ii) A subtler corollary of consistency achieved consensus between 1979 and 1983 (Gurevitch & Isotomin 1979; Gray & Kilkenny 1980; Schoub 1983; Scudder & Olbert 1983). This test results from establishing the conditions for convergence of the perturbation expansion used when extracting the Spitzer–Braginskii Fourier heat law. Since this time, it has been widely agreed in the plasma literature that the Knudsen number of the Spitzer expansion must be exceedingly small for the Fourier heat law to be CJC defensible (for an accessible derivation see Scudder & Karimabadi 2013). Specifically, this condition is that

$$|\mathbb{K}_{P_e}| \leqslant 0.01, \tag{12}$$

implying that the mean free path for collisions must be less than 1/100th of the scale of the electron pressure gradient. For this reason, the Spitzer formulation is hardly ever physically appropriate for astrophysical plasmas where $\mathbb{K}_{P_e} \simeq O(1)$ are commonly expected (Scudder & Olbert 1983; Scudder & Karimabadi 2013) for $r > 1.05\,R_*$, where R_* is the nominal stellar radius.

After formulating a CR (part I) that enables a fluid solution to be found, it is then possible to determine retrospectively $\mathbb{K}_{P_e}$ and $\mathbb{K}_{T_e}$ and compare them with the two mathematical perturbation requirements above for consistency with the Spitzer–Braginskii heat law.

Below, the condition of Equation (12) is also shown to be contradicted by all four of our VoM case studies. The existence of this second test predated only the last two VoMs reviewed here. Nonetheless, all VoM efforts discussed below could have been contradicted using the careful statements of SH in their 1953 paper 66 yr ago.

(iii) VDF $\geqslant 0$. Either closure approach can fail by predicting negative values for the VDF within the speed domain required for the moments used Scudder & Karimabadi (2013).

The N-moment form of Equation (6) admits the possibility that the assumed, underlying VDF becomes unphysically negative. This is also possible with finite $\mathbb{K}_{T_e}$ with the CE expansion whose underlying VDF has a similar form: $f \simeq f^0(1 + g(\boldsymbol{v}))$, with a multiplicative factor $1 + g = (1 - \mathbb{K}_{T_e}\cos\theta(\alpha x^4 + \beta x^6))$, where $x = w/w*$ (p 243 Hazeltine & Waelbroeck 1998). As $\mathbb{K}_{T_e}$ increases, the speed-dependent corrections can potentially overpower unity making the approximated VDF unphysically negative, just as the tensorial contractions can do the same for the N-moment VDF.

The speed domain of this unphysical behavior moves toward lower speeds as $\mathbb{K}_T$ increases, eventually invalidating the structure of the VDF required for accurate moments used in the fluid conservation equations. These circumstances would remain undetected without a posteriori inspection of the occurrence of the negativity of the VDF for any VoM solutions generated with either Spitzer or the N-moment closures.

A reasonable FoM for the importance of $f < 0$ can be defined as follows. The evaluation is numerically performed for all moments used in the subsequent fluid equations and spaced at reasonable spatial increments to resolve the strong gradients of the moment solutions implied by the CR. The FoM($\boldsymbol{x}$) at $\boldsymbol{x}$ for a given spatial location is the *smallest* FoM found when inventorying all fluid moments required for the solution at the given spatial location using Equation (20). If the tabulated moment has no negative VDF values the moment FoM = 1; as the importance of VDF < 0 grows (as weighted by the phase space volumes they represent) the FoM decreases from unity. Different moments whose integrands maximize at larger thermal speeds have increased numerical sensitivities to decreasing the FoM and all must be checked. Thus, the heat flux would be impacted at weaker Knudsen numbers than the pressure tensor, or the momentum. (see Appendix B for an approach to computing FoM.)

Neither the CE nor Grad's N-moment CRs are a panacea for the difficulties of finite $\mathbb{K}$ plasmas involved in the corona below the sonic point. The nonnegativity check should be performed before making physical interpretations. This concern is especially important in regimes where the relevant Knudsen number is becoming non-perturbative and/or changing very rapidly, or in locales where local corroboration of the VDF underlying the closure may not be available for the foreseeable future. In the vicinity of 1.05–3 $R_\odot$, this problem is expected to be especially severe for the foreseeable future.

(iv) VDF $\simeq$ VDF$_{\rm obs}$ A reasonable expectation is to extend the N-moment method or CE radial domains to where there is knowledge of the ambient VDF to ascertain how well the measured VDF is anticipated by the assumed closure. (This might be done using the innermost petals of *Parker Solar Probe* or *Solar Orbiter* trajectories.) At the radial location where boundary conditions on the N-moments are imposed, a minimal consistency check would be to compare the eVDF$_{\rm theory}$ implied by the moment method and its closure technique, with eVDF$_{\rm obs}$ as measured in situ, but nearby. As an example, throughout the presently measured solar wind the eVDF is known to be nonthermal of the core-halo or kappa types with occasional strahl deformations. If the N-moment method at its closest point to the accessible solar wind observations cannot recover this type of eVDF, this is an indication of excessive rigidity.

As these are steady-state solutions outside the critical point, a boundary condition on the fluid moments could be at 1 au as considered by Demars and Schunk, in Figure 1(k).

Figure 1 contrasts 1 au eVDF portraits from observations and theory when the theory is initialized at 1 au. The (L) insets from observables produces the same moments with a bifurcated core-halo shape, while the (R) inset from 16-moment theory suggests an undifferentiated elliptical shape with skewness. Despite the 16-moment method's attempt to include more moment structure at larger Knudsen numbers, it still does so under the controlling influence of the central Maxwellian shape assumed for f^0. The bifurcated observed shape seen in the core-halo data of (L) is a key recovery of the new approach for electron solar wind transport Scudder (2019b).

Like the fluid equations based on Spitzer closure the N-moment method determines spatial and temporal variations of these moments as solutions of a larger set of coupled partial differential equations. The leverage of the approach (its closure) exploits a supposition that a theoretical velocity distribution function (Equation (6)) can adequately support the desired evolution of the moments. While it is obvious that the moments do not uniquely characterize the supporting velocity space in a 1–1 manner, it should also be clear that the value of the N-moment method could be inventoried by how well its mathematically dictated form replicates the observed VDF seen for the systems being analyzed. There is every possibility that the method of *moments* with correct *moment* boundary conditions will not correctly predict the system's eVDF behavior away from the boundary; this is unavoidably correct if the moment boundary conditions imply the underlying assumed VDF do not replicate the observed VDF precisely where the boundary conditions are imposed. Although the boundary conditions imposed may agree with observed moments, the prescribed VDF required by the N-moment method may be too rigid to *agree* favorably with observed VDF at the same boundary.

Another effect of the non-uniqueness of the supported VDF for a given set of moments is that moment "equivalent" VDFs can have markedly different collision integrals, which may or may not be important for the physical predictions of the N-moment approach. An important example of this circumstance is the thermal force that observationally does not get smaller with lower collisionality (Scudder 2019a) and is important in zero current regulation of the steady-state wind, but determined by the low-energy distortions of the eVDF that attend the flow of heat.

4. Coronal Case Studies: CJC

Four case studies are now examined retrospectively where CRs used without CJC have strongly influenced currently held paradigms of coronal solar wind physics. The first is about the coronal "heating" problem. The second is about the sufficiency of Parker's thermal wind model (Parker 1958) to explain the observed solar wind. The third and fourth examples showcase the status of VoM modeling for post Hartle–Sturrock (HS; Hartle & Sturrock 1968) wind models that are nearly contemporaneous with this writing. By picking studies sprinkled over the 80 yr development of this field, the generality, persistence, and misunderstandings of the closure problem are highlighted, and the need becomes clear for demanding CJC validation for fluid modeling of VoMs in the future.

4.1. (i) The Interpretation of Non-monotonic T(r)

Just after spectroscopic inferences of the presence of high coronal temperatures were made (Edlén 1937; Grotrian 1939), Alfvén published (Alfvèn 1941) the inferred non-monotonic temperature profile of the corona shown in Figure 2–(*L*). This inferred temperature profile was based on Alfvén's model of force balance and observed electron density profiles, but did not rely on a closure postulate. Many profiles since this time have shown similar non-monotonic profiles (see, e.g., the suite collected in Lemaire & Stegen 2016). Panels (L)–(M)–(R) of Figure 2 show radial profiles of temperature, pressure, and Knudsen numbers ($\mathbb{K}_P$ and $\mathbb{K}_T$) and mean free path information versus radius under two different assumptions: red (ignoring) blue (modeling) average magnetic effects from the same density observations (Alfvèn 1941).

From their earliest reports, these and similar data have been summarized as exhibiting the "coronal heating problem." The data's characterization comes from the argument: for the non-monotonic profile to be steady and consistent with thermal conduction, the observational profile requires heat to be deposited at the temperature maximum, r_m.... It is difficult to find articles in the present decade's coronal literature that do not refer to this characterization as established fact. Multiple research programs are presently focused on VoMs designed to produce this surmised mode of energy deposition.

This type of argument requires a remote knowledge of $\nabla \cdot \boldsymbol{q}$, a model-independent term in the plasma energy equation. The argument appears to assume that the plasma heat flow at coronal temperatures and densities obeys a Fourier's law form, just like in neutral gases:

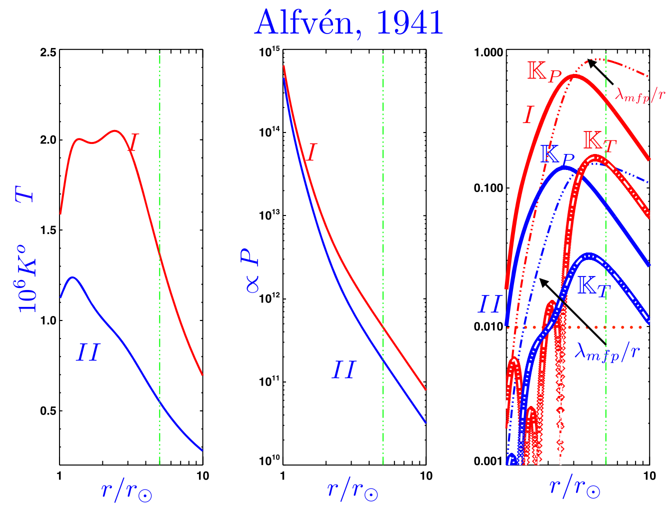

Figure 2. Alfvén's inference of the non-monotonic temperature profile of the solar corona (Alfvèn 1941). Models I and II are with and without models of assumed magnetic effects. (L): temperature; (M): pressure; (R): pressure Knudsen number, $\mathbb{K}_P$, and mean free path over radius: $\lambda_{\rm mfp}/r$, based on densities used to extract temperature profiles. Density observations most accurate below 5 $R_\odot$ (vertical green dotted line). Note the widespread violation of Spitzer's maximum allowed $\mathbb{K}_P < 0.01$. Also $\mathbb{K}_{T_e} < \mathbb{K}_{P_e}$ implies Spitzer's thermal force electric field is exceeded by the pressure gradients.

$$\boldsymbol{q} = -\kappa(T)\nabla T. \tag{13}$$

When Alfvén's profile was first exhibited, this Fourier law had not been derived for a *plasma*, making this type of argument one probably based on an adopted closure, since Chapman and Cowling had recently, in 1939, succeeded in deriving just such a relation for neutral gases (Chapman & Cowling 1939).

Since the heat flux enters the energy equation via the $\nabla \cdot \boldsymbol{q}$, its sign determines whether it heats or cools the plasma; evaluating this divergence at the observed maximum temperature (with attention to the functional dependence of the diffusivity in Equation (13)) yields:

$$\nabla \cdot \boldsymbol{q} = -\frac{d\kappa(T)}{dT}\left(\frac{\partial T}{\partial r}\bigg|_{r_m}\right)^2 - \frac{2\kappa}{r}\frac{\partial T}{\partial r}\bigg|_{r_m} - \kappa\frac{\partial^2 T}{\partial r^2}\bigg|_{r_m} > 0. \tag{14}$$

With $\partial T/\partial r|_{r_m} = 0$ and the negative curvature of T at the inverted temperature maximum, the sign of the divergence is indeed positive at r_m.

While the *mathematics* for the inequality in Equation (14) is correct, its *physical* interpretation is entirely contingent on the physical justification of the assumed heat law's form in Equation (13) at r_m. As an example, had the heat flux CR depended on the density gradient as well as the temperature's, (Scudder & Olbert 1983), the calculus would not have supported the heating conclusion summarized above.

The Spitzer–Härm 1953 derivation (Spitzer & Härm 1953) of a Fourier heat law for a collisional plasma seemed to bolster its use to interpret the coronal temperature profile. Braginskii's 1965 announcement of a similar form (Braginskii 1965) seemed to cement the rationale for using such a form and the heating inference summarized above.

However, tucked in the 1953 SH derivation there is the explicit zero current condition, summarized as Equation (7) above, and the assumption that the Fourier law was a perturbative result in a nearly isobaric background plasma. The radius of convergence of this derivation was established by 1980, Equation (12). Until the plasma profile is shown to be at least consistent with both of these prerequisites, it is inconsistent to pursue consequences of Spitzer's heat law form for its implications about the temperature maximum.

Referring to Figure 2(R), we can *evaluate* whether Spitzer's heat law form is appropriate for the interpretations ascribed to this data. Under both of Alfvén's red or blue assumptions

$$|\mathbb{K}_{P_e}|_{\mathrm{Alfvén}} > |\mathbb{K}_{T_e}|_{\mathrm{Alfvén}}, \quad (15)$$

which implies that Spitzer' zero current condition embedded in his heat diffusivity and Fourier law form is *contradicted*. Not unexpectedly, but clearly evident in the Alfvén profiles is the fact that $|\mathbb{K}_{P_e}| > 0.01$, showing that Spitzer–Braginskii perturbative closure does not converge for the Knudsen number regime at r_m.

Neither of these lines of reasoning were available to Alfvén. However, since SH's 1953 paper, the original characterization about the nature and cause of the non-monotonic temperature profile is inappropriate. Where the energy *must* be supplied is clearly no longer established by this argument, despite the number of published VoMs being designed to support the original, but incorrect, inference of the maintenance of the temperature maximum.

The English language provides different ways to refer to the non-monotonic temperature puzzle of the corona: perhaps the least warranted description is that it reflects heating; descriptively, it is a temperature *inversion*, Scudder (1992). To advance an explanation for the temperature inversion of the corona requires a deeper understanding of the physics of these layers than to interpolate that it has been *heated* there.

4.2. (ii) Thermally Driven Winds: Two-fluid Interpretation

A different variant of this same kind of imprecision occurred when a two-fluid *thermal* model of the coronal expansion was published in 1968 by Hartle and Sturrock, (HS). Based on coulomb collision scattering physics and the then recent Braginskii transport equations (Braginskii 1965), a two-fluid description of the solar wind was numerically generated from equations closed using Spitzer–Braginskii Fourier heat laws particularized for protons and electrons. The coupling between the two scalar pressure fluids retained a one-fluid momentum equation but retained separate electron and ion energy equations with collisional energy exchange rates (modeled as between Maxwellian VDFs) as systemized in Braginskii's influential paper (Braginskii 1965).

Upon integration from accepted coronal base conditions, the model generated a supersonic wind but could not produce the highest 1 au solar wind speeds known at that time, nor the proper partition between the electron and ion temperatures, nor the observed size of the 1 au heat flux.

After presenting their new solar wind solution and comparing it with available 1 au data, HS vacated their own thermal model for the wind in the same paper Hartle & Sturrock (1968), suggesting that the model-data contrasts implied their thermal model lacked essential missing physical ingredients of the corona/wind problem. They speculated on the likely

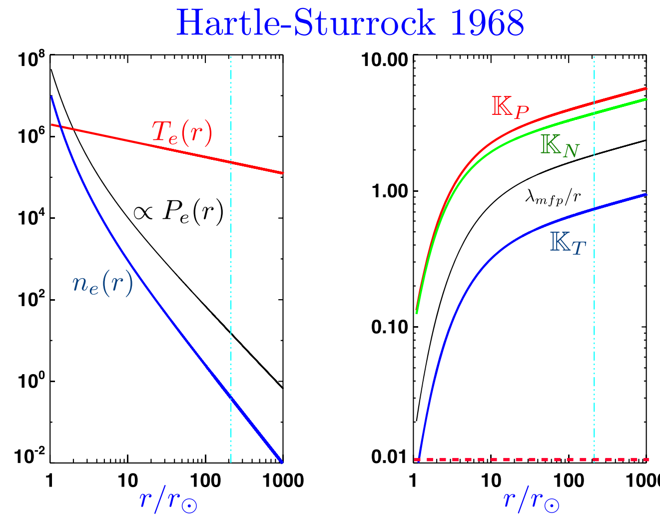

Figure 3. Left panel: T_e, P_e, n_E, vs. r in red, moss, blue. Right panel: variation for HS solar wind solution (1968) of Knudsen numbers for P_e, n_e, T_e, together with mean free path over radius in red, green, blue, and black. Note: solution characterized by $\mathbb{K}_P > \mathbb{K}_T$ and all values exceed the value at the horizontal dashed red line which is the *upper bound* (Equation (12)) for valid Spitzer–Braginskii closure. Accordingly, the heat law closure is shown to have been invalid throughout the solution!

candidates for additional mechanisms, suggesting the damping of hydromagnetic waves or wave pressure acceleration effects might explain the *exhibited* data-model deficits.

Constructed in 1966–1967, the HS model had incorporated results of Braginskii's 1965 paper and Spitzer's 1953 derivation of a plasma heat law of the form in Equation (13). There is, however, no mention in the HS paper about the condition above of Equation (7) that occurs as Equation (25) in SH. The HS solution could have been evaluated a posteriori by testing their profile's compatibility with the relation SH had assumed, viz

$$\mathbb{K}_{P_e}^{SH} = -0.71\mathbb{K}_{T_e}^{SH}. \quad (16)$$

A quick perusal of the Knudsen profiles in Figure 3(R) illustrates that Equation (16) is contradicted since $\mathbb{K}_{P_e} > \mathbb{K}_{T_e}$ everywhere within the HS two-fluid solution. When writing their paper, HS could have made use of this point to understand the fundamental problem: their solution used the Spitzer closure beyond the domain where the electric field was given by Equation (7) found necessary by SH to accompany their perturbative Fourier heat law. Alternately, violating this condition implies the CR of SH had been used inconsistently in gradients that were not perturbative.

Although HS discussed the strong variability of the collision frequency across their solution, they concluded that their fluid framework was reasonably secure well beyond the orbit of earth. HS did *not* examine a posteriori the variation of their solution's pressure Knudsen number, $\mathbb{K}_P(r)$, shown in Figure 3(R) computed for this paper. This figure also shows that $\mathbb{K}_P$ exceeded the horizontal red dashed line known since 1983 to be the upper limit for a convergent perturbation expansion for Spitzer's heat law.

Figure 3 shows that the HS thermal wind solution was closed inappropriately for the entire radial domain for which their solar wind solution was determined, a fact HS could have known about; in addition, but unknown to them, they had inappropriately used

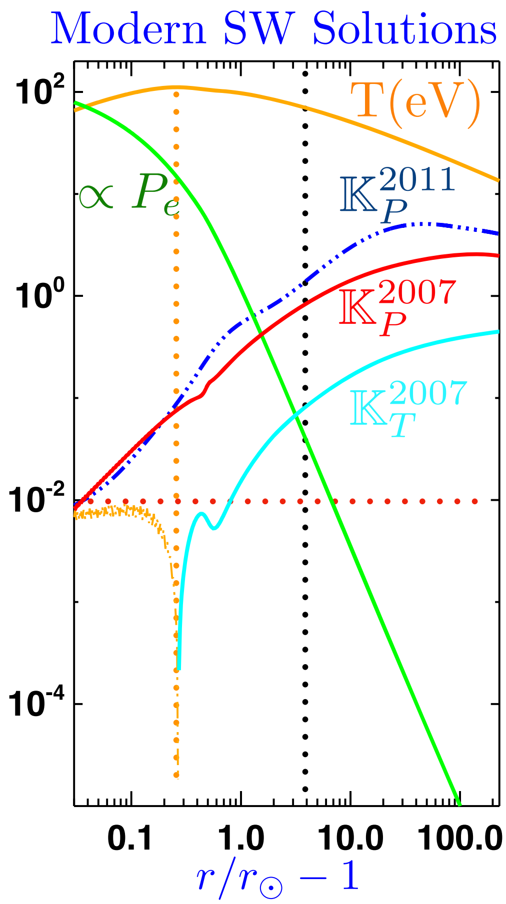

Figure 4. Profiles from recent solar wind solutions (Chandran et al. 2011; Cranmer et al. 2007) of $T(eV)$, P, $\mathbb{K}_P^{2011}$, $\mathbb{K}_P^{2007}$, in orange, green, blue, red, respectively. The horizontal dotted red line is upper bound on $\mathbb{K}_P$ where Spitzer–Braginskii is valid. The vertical black dotted line where heat conduction *bridging formula* converts from SH to Hollweg saturated form. The solution's inferred temperature maximum is indicated by the orange vertical dotted line; it is well within the domain where SH closure dominates the bridging formula where it is assumed valid, but is not. This region is also where the wind is predominantly accelerated and has been modeled in both solutions as if SH closure were appropriate, which it is not. This may be seen by the red dotted horizontal line at 0.01, which is the upper-limit Knudsen number allowed for a convergent SH closure. Both solutions fail this test. The orange-cyan curve for $|\mathbb{K}_T|$ shows a cusp at the location of the solution's temperature minimax. Data from the solutions analyzed were digitally shared with the author by the principal authors of each study (B. A. Chandran 2012, private communication; S. R. Cranmer 2012, private communication).

the Fourier heat law when $\mathbb{K}_{P_e}(r) > .01$ that was outside its subsequently derived radius of convergence.

Careful study of SH could have warned all researchers since 1969 that the Hartle–Sturrock model data confrontation and conclusions were flawed. The results in 1983 should have sounded a secondary warning that the results of HS did not have the leverage ascribed to it in nearly every solar wind theoretical paper since that time. This situation shows a research program reticent to winnow its established theoretical basis in the presence of new research as embodied in the Spitzer-Härm work and the results about convergence of Spitzer's CR solidified in 1983.

Curiously, HS had argued (p 1164) that their heat flow closure was satisfactory out to the 1 au locale of model data comparisons, since they felt it was only in question beyond 5 au, where the model's heat flux approximated the "saturated" value. The saturated form of the Fourier heat law corresponds to temperature Knudsen number $\mathbb{K}_T \simeq O(2/3)$. Their argument of closure sufficiency appears to have presumed the size of $\mathbb{K}_T$ *alone* ensured the viability of the Fourier closure. However, the *spirit* of the CE approach that allows the heat law derivation clearly supposes that all Knudsen numbers associated with gradients from the left-hand side of the kinetic equation should be perturbatively small; hence, the condition for the expansion is clearly on the pressure Knudsen number which exceeds that of T_e, n_e and $|U|$, since

$$|\mathbb{K}_P| \equiv |\mathbb{K}_n + \mathbb{K}_T| \simeq |\mathbb{K}_n|(1 \pm \epsilon); \quad \epsilon \ll 1, \tag{17}$$

because the temperature scales are so much larger than the density everywhere.

Since 1968, HS's conclusion (Hartle & Sturrock 1968) that new effects are required beyond the treatment of the "thermally" driven wind should also have been found logically indefensible, since the closures HS used to provide model-data disagreements were inconsistent with the premises of the Spitzer CR used at all radii.

This retrospective also implies that HS's simulation and 1 au data-model deficit did not logically comment on the viability of a physical, defensibly closed, thermal wind model recovering the 1 au observations. In the intervening time, evidence to support the existence of alternate thermal solutions has been presented for hybrid-kinetic solar wind solutions that circumvent closure requirements and improve the description of the coulomb physics; these have produced high-speed winds with observed properties (e.g., Olbert 1983; Landi & Pantellini 2003) without including wave phenomena that HS surmised were necessary for this purpose. Even exospheric solutions that are more properly closed with respect to the size and effects of the necessary $E_\|$ also show promise in this direction, e.g., Maksimovic et al. (1997).

As of this writing, HS's 1968 conclusions recur frequently in the literature to justify undertaking many investigations exploring additions to the coronal-wind description beyond that contained in the thermal model. Two of these will be briefly examined next; they include direct acceleration by waves and heating by turbulence.

4.3. (iii) Wave-driven Winds: One-fluid Interpretation

An example of a 21st century coronal solar wind modeling addressing the HS surmise, is found in the extensive one-fluid model (Cranmer et al. 2007) that included non-WKB wave heating, wave pressure driving, and models of turbulent cascades. This effort was said to be a "self consistent treatment"; nonetheless, it contained an inconsistent moment closure.

Apart from addressing new effects, this solution was only made possible by a closure interpolation arranged to be identical with Spitzer and Braginskii below $5\,R_\odot$ and smoothly transitioning to one suggested by Hollweg (Hollweg 1974) beyond $r > 5\,R_\odot$. This merging of the two heat law forms left the Spitzer Fourier law form dominant below $5\,R_\odot$. These pre-existing closures were used without modifications that might have been necessitated by the introduction of new electrodynamic processes in the modeling that impact maintenance of zero current. This CR was used, without retro-validation in the spirit of CJC.

Self consistency would ordinarily imply "...not having parts or aspects which are in conflict with each other.." The HS thermal solution above was shown a posteriori to be self contradictory of its Spitzer CR. Has this interpolating approach to the heat flow closure in this 21st century coronal-wind model overcome this difficulty when using Spitzer CR in the inner region? Does using essentially the same CR as used by HS get any more "self consistent" here when used in the 21st century VoM with an added spectrum of new mechanisms superposed?

The new processes involved in this VoM were introduced for their impact on the solutions properties. In principle, many things could change for the plasma modeled as a fluid. The question of force balance and maintenance of zero current in either limiting form of the interpolating closure, were not considered when bridging between the two existing closures.

For this 21st century coronal-wind modeling exercise, Figure 4 shows that the proffered VoM with Spitzer CR violated (i) Spitzer's choice of parallel electric field since its $\mathbb{K}_P^{2007}(r) > \mathbb{K}_T^{2007}(r)$, and (ii) violates the convergence requirement of Spitzer $\mathbb{K}_P^{2007}(r) > 0.01$, given by the red dotted line established in 1983. The vertical black dots denote where this 2007 modeler *chose* to make a cross-over ($5\,R_\odot$) between Spitzer dominated (below) and Hollweg saturated heat flows (above) for this *self-consistent* treatment.

For reference, the orange vertical dotted line indicates the location of the temperature maximum of this VoM solution. Among other things, this solution perpetuates Spitzer's suggestion that the heat goes down temperature gradients well up into the regime where the *form* of the heat law is itself suspect. In turn, this choice implies the transition region is a significant energy drain for the plasma near the temperature maximum, impacting the energetics of its formation. This is just one of the ways that the incorrectness of the closure can circumscribe the remaining requirements that the VoM must produce to agree with observations. The accuracy of the CR used will and can influence the perceived importance of any given VoM.

Since the solar wind's acceleration is largely completed by 5–10 $R_\odot$ this solution is attempting VoM modeling of the wind's acceleration, while simultaneously closing the fluid equations with a broken description of heat flow in this inner acceleration region $h < 4\,R_\odot \leftrightarrow r < 5\,R_\odot$. It is hard to understand such modeling as self consistent, since without a physically defensible closure truncation, there is no viable, zero current, fluid plasma description with which to study new effects. In addition inaccuracies of the divergence of q that arise from the broken closure also impact the efficacy with which heat is converted to flow energy and the perceived need for additions of energy in other forms.

The closure, for all of its new effects, is inconsistent with zero current at the Generalized Ohm's Law level and steady state and is thus vacated using the 1953 framework of the SH paper. As expected, the size of $|\mathbb{K}_{P_e}|$ in this 2007 VoM solution exceeds (almost everywhere) Spitzer's allowed upper limit of 0.01, known since 1983. On both accounts this "self consistent" work with over 340 citations, actually has no physically defensible conclusions.

4.4. (iv) Wave-driven Winds: Two-fluid Interpretation

Another 21st century two-fluid solar wind model has examined VoM modeling including ion kinetic effects and low-frequency non-WKB wave-driven winds, including the interdiction of proton anisotropy evolution (Chandran et al. 2011). It has attempted to include additional fluid averages of kinetic effects into the two-fluid equations, so that their influence on the subsequent evolution of the wind may be evaluated. In this sense, this modeling might be considered as a recent attempt to improve the 43 yr old HS two-fluid solution (Hartle & Sturrock 1968), by adding new effects HS had speculated were necessary. Compatibility with steady-state quasi-neutrality and zero parallel current were enforced by inserting $n_e = n_i = n$ and $\boldsymbol{U}_e = \boldsymbol{U}_i = \boldsymbol{U}$ into their initially two-fluid treatment of the plasma fluid system of equations. Coulomb collisional coupling of energy was retained based on Maxwellian ions and electrons; ion energy equation was closed with a fourth moment heat law CR (Snyder et al. 1997) assuming that the ions were nearly bi-Maxwellian in the drift frame. A postulated high-frequency wave particle scattering process was presumed to moderate the ion pressure anisotropy. The electron fluid closure incorporated a heat law form virtually identical with the bridging formula for the electron heat law used in the modeling of case III above (Cranmer et al. 2007). The thermal force was not included in the respective energy equations.

An a posteriori analysis of this VoM is displayed by the curve labeled $\mathbb{K}_P^{2011}$, shown with dark blue curve in Figure 4. Because the heat-law closure adopted for electrons was intended to reduce to Spitzer–Braginskii to the left of the vertical black dotted line in this figure, this domain of the solution is fairly assessed a posteriori by requiring the Knudsen number to be below the accepted maximum size 0.01 allowed for convergence of the Spitzer transport recipe. As shown in this figure, the entire inner heliospheric solution (dark blue curve) is improperly closed in this manner. The assumptions for the heat conduction recipe are contradicted beyond $r/R_\odot - 1 > 0.05$, with Knudsen numbers exceeding the maximum allowed for Spitzer indicated by the horizontal red dashed line.

With an unphysical underlying closure across the entire inner heliosphere where the wind is principally accelerated, what is the take-away from this VoM effort that includes new physics? Are the waves included shown to be essential or accidental? By improving agreement with data does this fluid solution verify the wave-driven VoM wind model? Because (i) the closure is required for the fluid description integration and (ii) is demonstrably broken, this VoM unfortunately does not allow an unassailable, or even likely, physical VoM result. Nonetheless, this work is widely emulated and often cited for its closure approach.

5. Discussion: Solar Probe Assays

5.1. VoMs and Closure Headaches

At present, all published VoMs are susceptible to being invalidated because they represent only the outcomes of a provisional hypothesized CR, with no CJC control of its validity. As discussed in the two most recent case studies, CRs are increasingly implemented as hybrids of CRs thought to be appropriate in different radial regimes of some other plasma fluid problem. Among the difficulties of these and other approaches (e.g., Breech et al. 2009; Cranmer et al. 2009) is the omission of the thermal force physics Scudder (2019a) required for two-fluid descriptions with separate energy equations. More generally, the total absence of a posteriori CJC cross checking of the closure hypotheses *using the solution produced* gives no assurances that the VoMs produced are consistent descriptions of reality.

The conclusions generated by such *unproofed* closures have demonstrably deflected coronal research directions for multiple decades because their physical consistency was neither challenged (i) before their conclusions were accepted for publication, nor (ii) when new knowledge gave retrospective clarity for previously made pivotal assumptions. Unchallenged, they are presently ensconced as deeply held, but still inappropriate, rationales for new coronal solar wind interpretations by a new generation of researchers. The long-standing, but partially forgotten, thermal force physics is only now being realized to be a significant part of the required VoM modeling for the present coronal and solar wind challenges.

This paper has emphasized the idea that the CR is only the *first* part of the closure approach, and by itself can only give a *conjectural* guidance for what the approximate fluid behavior of the VoM *could be*. Given the nonlinearity of the equations involved and the strong gradients involved in the corona and inner solar wind, the *validation phase*, CJC, of the "closure process" is the needed screen for the very real situation that the CR alone has produced results with no *self-consistent* physical basis. This inconsistency may include being "prominently supported" by VDFs with negative probabilities in them. When this happens, the rigorous fluid equations are conserving particles while anti-particles implement zero current and conservation of energy. Appendix B of this paper shows an efficient way to screen fluid solutions for the occurrence of significant negative VDF behavior. In the present epoch, the neglect of including the thermal force when explaining differential heating signatures is a serious criticism of the relevance of a proposed VoM, since the thermal force and heat flux are a tandem (Fitzpatrick 2014; Scudder 2019a) that promote exchange of energy consequences (Scudder 2019a).

From this precaution, it is clear that another test for a proposed VoM plasma fluid closure recipe is to ascertain the specification of its underlying VDF predicate. *Ad hoc* moment closure recipes have no supporting VDF formulation that implements the CR recipe assumed. As an example, the interpolating CR between Spitzer and Hollweg limiting forms (in studies 3 and 4 discussed above) never declares how the VDF transitions in space to smoothly produce the heat flux CR interpolations supposed. Studies that include empirical heat flux profiles (Breech et al. 2009; Cranmer et al. 2009 to avoid the problems of a theoretical closure have this same difficulty. By contrast, CE and N-Moment closures have clearly stated underlying VDFs that are their spatially dependent connections to the kinetic equation and the basis of their spatially dependent, nonnegative VDF screens in Appendix B. VoMs using CRs without a specification of the VDF predicate preclude tests for negative VDF allowed by the FoMs of Appendix B. Such VoMs have a cloud of ambiguity over their conclusions for this reason, if for no other.

The internal contradiction of a given CR is clearly possible when used for a different physical system, despite the very same CR having been CJC validated for a different system. The CJC approach emphasizes the validation part (ii) of the closure treatment involves tests on the fluid solution for the specific plasma system under study; it cannot be validated globally in the CJC sense by illustrating some other plasma system where the CR satisfied CJC scrutiny. The CR must be validated as a function of space across the entire volume of the VoM, since it may be viable only in part of that volume. To do so requires the fluid solutions enabled as a function of space and the reconstruction of the underlying VDF and the methods of Appendix B. Without these CJC tests, the VoM may be interpreted as representative of the entire volume when it is not.

As widely and as long as it has been known that the Spitzer-Braginksii CR has a severe convergence requirement, there has been virtually no attention paid to it in the coronal and solar wind literature. These CRs are the choice of almost all who would model the solar corona and wind between $1 < r < 5\,R_{\odot}$, but apparently justification of this choice is not worth any ink! As discussed above, *N*-moment methods have been tried with limited successes; their implied VDF functional forms determined by the moment solutions need checking for nonnegativity. Although less frequently in use, the *N*-moment methods still require an analogous two stages for their closures: CR followed by validation that has never been implemented.

The currently used CR "only" approaches (without CJC) bear a very strong resemblance to an attitude explained to the author many years ago (M. L. Goldstein 1980, private communication), called WECIDO. That is to say, the CR used is excused by the absence of alternatives: "..What Else Could I Do (WECIDO)..." It is a bad trade, since with such a point of view all responsibility for the precision of the closure is avoided, the simpler fluid problem is now closed by assumption, and the implied fluid solutions can be exhibited; but what do they mean? The adopted CR has made suppositions; the solutions produced with this adopted CR can be retroactively analyzed, as advocated above. In the present environment, it should not be surprising that this is not done.

The WECIDO'ers go on to interpret their CR enabled fluid solutions as if they *must* have physical implications. And, if the fluid solutions do not explain observations, they *must* imply that the VoM being tested is intrinsically incomplete physically, a haunting echo of the incorrect self-interpretation by HS. Or, if they do explain observations the mechanism of the VoM is supported. Both conclusions are logically indefensible.

WECIDO logrolling is another prevalent pattern. Adopting a previously published part (I) CR appears to earn a "community pass" that its fluid solutions are publishable VoM efforts—even when the recourse to that Spitzer CR is long since known to be a WECIDO excuse. With a wide group of practitioners *presently* adopting the same CR (as in case studies 3 and 4 above) *without attempting validation*, there seems to be a wagon circling defense of the indefensible (Mikic et al. 1999; Usmanov & Goldstein 2006; Cranmer et al. 2007; van der Holst et al. 2010; Chandran et al. 2011; Manchester et al. 2012; van der Holst et al. 2014; Gombosi et al. 2018; Réville et al. 2018). In 2018 these "thermodynamic" VoMs were reviewed with the assay: While this "..."thermodynamic" approach sidesteps [sic!] the underlying physics of coronal heating and solar wind acceleration it provides an adequate mathematical framework [sic!] to describe coronal processes in a way that is consistent [sic!] with solar dynamical processes" Gombosi et al. (2018); emphasis added. WECIDO is still alive, well and applauded. We are now more than 50 yr past the learning period for two-fluid coronal VoMs, and something better than a verbally adroit form of WECIDO is required to make coronal and solar wind VoMs useful for advancing the physics of these layers.

5.2. Avenues for Closure Validation

Two clear a posteriori profile tests have been suggested that would pre-screen (CJC) fluid plasma VoM attempts using Spitzer–Braginskii closure. By our retrospective sample, it seems hard to foresee that such closures will produce physically defensible VoM demonstrations inside 5–10 $R_\odot$. A third a posteriori test has been suggested for CE class and Grad *N*-moment closure approaches involving assays for positivity of underlying VDFs. A fourth avenue was suggested using the radial domain of observations overlapping the radial domain of the VoM. A fifth screen involving the presence of the thermal force was argued to be a general requirement for the VoMs that require different energy equations for plasma species.

When using new variants of either CE or *N*-moment closures, a posteriori FoM checks should be used to demonstrate that the underlying VDF of the closure remains "substantially" nonnegative throughout the velocity space involved and across the spatial domain of the VoM attempted. This precaution is designed to preclude moment profiles based on conservation of particles using anti-particles (VDF < 0) in the book keeping, or zero current with positrons posing as electrons, (anti-protons as protons) or both in any other moment such as the heat flux being algebraically misinventoried in the presence of VDF <0. Appendix B develops a systematic way to formulate a FoM for the occurrence of this velocity space defect underneath the fluid solutions; unphysical fluid behavior determined by the CR may otherwise be taken as physical.

At the same time, this approach is cognizant of the fact that *all* CE and *N*-Moment VDFs *eventually always* go negative, but usually do so at very high speeds for perturbative Knudsen numbers where they make negligible contributions to the moments. Such limits provide a nice check on the diagnosis of FoMs for such negativity. With the finite Knudsen numbers seen in the solar wind, the underlying eVDF supporting Spitzer's closure is negative within the speed range that impacts the numerically convergent heat flux and pressure anisotropy.

5.3. Parker Solar Probe *and* Solar Orbiter

Those seeking the prized VoM for the coronal temperature inversion or the solar wind's range of accelerations should first prepare a CJC specific to their proposed VoM. Particularly demanding will be its demonstration that specialized VoM effects that interact preferentially with ions rather than electrons (or vice versa) are analyzed as part of the new closure, and that force balance is demonstrably achieved in the ion frame, under the new CR. In this regard, the modifications of the *unavoidable* thermal force description must also be included. The consistency requirements for a viable closure approach for $1 < r < 5\,R_\odot$ and the successful VoM explanation would appear more arduous than any performed to date and certainly will not be believably performed using weak gradient Spitzer–Braginskii closures. This suggests that fluid plasma VoMs in the *Parker Solar Probe* and *Solar Orbiter* era will probably require different frameworks from those closely patterned after regimes with successful neutral gas transport descriptions that are currently in use.

The helpful referees are thanked for their comments. It is a pleasure to acknowledge extensive conversations with the late R.E. Hartle, NASA-GSFC, and discussions with S. Baalrud at the University of Iowa. The energetic support for new thrusts in research by NASA-SMD is sincerely appreciated. Editorial comments by SED are gratefully acknowledged. Partial support of this research was provided by NASA grant 80NSSC19K1114.

Appendix A
Acronyms and First Use

Shown in alphabetical order are the acronymns used that are most common in the literature.

BCR ↔ borrowed closure rule, Section 1.
CE ↔ Chapman–Enskog, Section 2.
CR ↔ closure rule, Section 1.
CJC ↔ Completely Justified Closure, Section 1.
eVDF ↔ electron VDF, Abstract.
FoM ↔ Figure of Merit, Section 3.
HS ↔ Hartle–Sturrock, Section 4 .
iVDF ↔ ion VDF, Abstract.
LTE ↔ Local Thermodynamic Equilibrium, Section 2.
SH ↔ Spitzer-Härm, Section 2.
VDF ↔ velocity probability distribution function; if physical $\geqslant$0, Abstract.
VoM ↔ Verification of Mechanism, Abstract.
WECIDO ↔ What Else Could I Do? Section 5.

Appendix B
FoM for Nonnegativity Impact on Moments

Assume a 3D velocity space indexed as $\ell \leftrightarrow i, j, k$ that is sufficiently dense in v_i, θ_j, ϕ_k to allow the desired moments to be determined well numerically as a weighted triple sum with quadrature grid weighting Q_ℓ. At this resolution, inquire the sign of the underlying VDF that supports the closure VDF solution denoted f_ℓ; construct indexed Heaviside like matrices with the properties:

$$\begin{aligned} P_\ell = 1 &\leftrightarrow f_\ell \geqslant 0 \\ P_\ell = 0 &\leftrightarrow f_\ell < 0, \end{aligned} \tag{18}$$

and

$$\begin{aligned} N_\ell = 0 &\leftrightarrow f_\ell \geqslant 0 \\ N_\ell = -1 &\leftrightarrow f_\ell < 0. \end{aligned} \tag{19}$$

For a given moment $\mathcal{M}$ with velocity space weights $\mathcal{M}_\ell$ at the mesh points where f is tabulated, the FoM for nonnegativity is

$$\mathrm{FoM}(\mathcal{M}) = 1 - \frac{\Sigma_\ell f_\ell N_\ell \mathcal{M}_\ell Q_\ell J_\ell}{\Sigma_\ell f_\ell P_\ell \mathcal{M}_\ell Q_\ell J_\ell}, \tag{20}$$

where $\ell \leftrightarrow i, j, k$, and J_ℓ is the velocity space Jacobian. The FoM($\mathcal{M}$) is exactly unity when $N_\ell = 0$ for all ℓ. The FoM then reflects the fraction of that moment determined by nonnegative f values.

ORCID iDs

J. D. Scudder https://orcid.org/0000-0001-7975-5630

References

Alfvèn, H. 1941, Arkiv fur Mathematik, Astronomie und Physik, 27A, 25
Bale, S. D., Pulupa, M., Salem, C., Chen, C. H. K., & Quataert, E. 2013, ApJ, 769, L22
Balescu, R. 1988, Transport Processes in Plasmas (Amsterdam: North Holland)

Braginskii, S. I. 1965, AdPlP, 1, 205
Breech, B., Mattheus, W. H., Cranmer, S. R., Kasper, J., & Oughton, S. 2009, JGRA, 114, A09103
Chandran, B., Dennis, T., Quaetert, E., & Bale, S. D. 2011, ApJ, 743, 197C
Chapman, S. 1916, RSPTA, 216, 279
Chapman, S., & Cowling, T. G. 1939, Mathematical Theory of Non-Uniform Gases (Cambridge: Cambridge Univ. Press)
Cranmer, S. R., Mattheus, W. H., Breech, B. J., & Kasper, J. C. 2009, ApJ, 702, 1604
Cranmer, S. R., van Ballegooijen, A. A., & Edgar, R. J. 2007, ApJS, 171, 520
Demars, H. G., & Schunk, R. W. 1991, P&SS, 39, 435
Edlén, B. 1937, Zeitschrift Physik, 104, 407
Enskog, D. 1917, Uppsala Inaugural Dissertation, Uppsala Univ.
Feldman, W. C., Asbridge, J. R., Bame, S. J., Montgomery, M. D., & Gary, S. P. 1975, JGRA, 80, 4181
Ferziger, J. H., & Kaper, H. G. 1972, Mathematical Theory of Transport Processes in Gases (Amsterdam: North Holland)
Fitzpatrick, R. 2014, Plasma Physics: An Introduction (Boca Raton: CRC Press)
Gombosi, T., van der Holst, B., Manchester, W. B., IV, & Sokolov, I. V. 2018, LRSP, 18, 4
Grad, H. 1949, CPAM, 2, 331
Gray, D. R., & Kilkenny, J. D. 1980, PlPh, 22, 81
Grotrian, W. 1939, NW, 27, 214
Gurevitch, A. V., & Isotomin, Y. N. 1979, JETP, 50, 470
Hartle, R. E., & Sturrock, P. A. 1968, ApJ, 151, 1153
Hazeltine, R. D., & Waelbroeck, M. L. 1998, The Framework of Plasma Physics (Boulder, CO: Perseus Books)
Hollweg, J. V. 1974, JGRA, 81, 1649
Landi, S., & Pantellini, F. 2003, A&A, 400, 769
Lemaire, J. F., & Stegen, K. 2016, SoPh, 291, 3659
Li, X. 1999, JGRA, 106, 8217
Lie-Svendsen, O., Leer, E., & Hansteen, V. H. 2001, JGRA, 106, 8217
Maksimovic, M., Pierrard, V., Lemaire, J., & Stegen, K. 1997, A&A, 324, 725
Manchester, W. M., IV, van der Holst, G., Tóth, G., & Gombosi, T. I. 2012, ApJ, 756, 81
Mikic, Z., Linker, J. A., Schnack, D. D., Lionello, R., & Tarditi, A. 1999, PhPl, 6, 2217
Olbert, S. 1983, in Solar Wind 5 NASA Conf. Proc., CP-2280, ed. M. Neugebauer (Washington, DC: NASA STI), 149
Pannekoek, A. 1922, BAN, 19, 107
Parker, E. N. 1958, ApJ, 128, 664
Réville, V., Tenerani, A., & Velli, M. 2018, ApJ, 866, 38
Rosseland, S. 1924, MNRAS, 84, 720
Rossi, B., & Olbert, S. 1970, Introduction to the Physics of Space (New York: McGraw-Hill)
Schoub, E. C. 1983, ApJ, 266, 339
Scudder, J. D. 1992, ApJ, 398, 319
Scudder, J. D. 2019a, ApJ, 882, 146
Scudder, J. D. 2019b, ApJ, in press
Scudder, J. D., & Karimabadi, H. 2013, ApJ, 770, 26
Scudder, J. D., & Olbert, S. 1983, in Solar Wind 5 NASA Conf Proc. 2280, 22, ed. M. Neugebauer (Washington, DC: NASA STI), 149
Snyder, P. B., Hammet, G. W., & Dorland, W. 1997, PhPl, 4, 3974
Spitzer, L. J. 1962, Physics of Fully Ionized Gases (New York: J. Wiley-Interscience), 144
Spitzer, L. J., & Härm, R. 1953, PhRv, 89, 977
Usmanov, A. V., & Goldstein, M. L. 2006, JGRA, 111, A7
van der Holst, B., Manchester, W. B., IV, Franzin, R. A., et al. 2010, ApJ, 725, 1373
van der Holst, B., Sokolov, I. V., Meng, X., et al. 2014, ApJ, 728, 81